Keeping Up with the Joneses as an Outcome of Getting Ahead of the Smiths. A Two-Stage Veblenian Status Game

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Abstract

In a status game, homogenous individuals first decide on their income (and on the effort necessary to that end) with the aim at *Getting ahead of the Smithes* (GAS). Next, they make use of a pure positional good to make incomes visible. Although the GAS hypothesis is ordinal, the signalling costs induce cardinal social concerns. The GAS hypothesis, translated into the KUJ (*Keeping Up with the Joneses*) (pride) concern, generates an equilibrium in which identical agents have unequal income levels. This equilibrium is an egalitarian optimum. But utilitarian and Paretian inefficiency are the price paid for equality.

Key words : Status game, Social concerns, Income inequalities, Conspicuous consumption, Well-being, Efficiency.

JEL Classification numbers: D3, D6, D8, I3, Z1.

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"So far as concerns the present question, the end sought by accumulation is to rank high in comparison with the rest of the community in point of pecuniary strength." - Veblen (1899)

"[...] *and the means of showing pecuniary strength and so of gaining or regaining a good name, are leisure and conspicuous consumption of goods."* - Veblen (1899)

1 Introduction

In this paper we build a simple two-stage game to account for the consequences of two behavioral assumptions made in the spirit of Veblen. In the first stage, homogenous individuals decide on their income (hence, on their effort to obtain that income) at the aim of reaching a high rank *in comparison with the rest of the community*. They are driven by what we refer to as Getting Ahead of the Smithes (GAS). This is the social competition game which explains financial accumulation in Veblen's *Theory of the Leisure Class* (1899). At the second stage participants decide on their conspicuous consumption (which has no intrinsic value) in order to signal their rank to society. As in Veblen (idem), conspicuous consumption is *the means of showing pecuniary strength*. The signalling game in which individuals engage provides a rationale for rich people consuming apparently useless goods.

The main insights of our contribution are as follows. First, the "indirect" utilities, deduced from the equilibrium of the signalling game, have a "Keeping Up with the Joneses" - KUJ, henceforth - "cardinal" form.[1] More precisely, according to these reduced utilities derived from the "ordinal" principle social concerns reflect the "pure pride" feeling. Individuals behave as if they were preoccupied by the gap between their own income and the average income in the population of lower ranked participants (Friedman and Ostrov, 2008). In other words, KUJ is an outcome of GAS . The intuition behind this finding is that signalling costs grow with the incomes of lower-ranked individuals. This result is welcome, since the KUJ version of relative concerns

[1] See Hopkins (2008) for a detailed distinction between the "cardinal" and "ordinal" versions of relative concerns.

sounds "ad hoc", whereas assuming that people worry about their income rank, the so-called GAS behavior, is quite plausible and indirectly confirmed by empirical papers in the spirit of Easterlin (1974). Second, the equilibrium distribution of income is non-degenerate. This result means that the desire for (advantageous) inequality is sufficient to explain income inequalities across *ex ante* identical individuals. The reason is that in a situation of income equality, an individual would find it profitable to get ahead of everybody by increasing his/her income.

Another noticeable result is that this unequal-income equilibrium is an egalitarian optimum in terms of utilities. This means that whoever dislikes true inequalities, that is inequalities in terms of utility, should like observed (gross) income inequalities, and should then oppose any redistributive tax policy. But true equality is costly. We show that there is a single utilitarian (social) optimum, relative to which all effort is lower than in the case of laissez-faire. As in Gavrel and Rebiere (2015), this is the consequence of a social rat-race phenomenon. But in our setup, the costs of making income visible (deduced from the signalling game) tend to moderate this distortion. This implies that perfect information can degrade welfare.

We believe that our contribution clarifies this line of research in the literature on status game. As the introductory quotes make clear, one cannot understand conspicuous consumption when incomes are known to everybody. Conversely, when incomes are public information, one can conceive a social competition game without conspicuous consumption. As Veblen points out, conspicuous consumption is valueless in a small community whose members know everything about everybody. In addition individuals do not necessarily feel any need to make their income rank known. Knowing that they are the "best" may be sufficient for them.

As far as we are aware, papers (closely) related to the present one restrict analysis to the signalling game (see for instance Hopkins and Korienko, 2004 and Ireland, 2001), which means that the distribution of incomes is exogenous. On the other hand, Gavrel and Rebiere (2015) restrict analysis to the social competition game, which *a priori* means that the ranks of individuals in the income hierarchy are assumed to be common knowledge.

Our investigation is organized as follows. Section 2 describes our setting and solves the signalling game. Section 3 shows how KUJ can be seen as an outcome of GAS and derives the equilibrium of the (reduced) social competition game. An analysis of the welfare properties of equilibrium is exposed in section 4, while section 5 extends the analysis to a more general setting. Section 6 concludes.

2 Environment and signalling equilibrium

In this economy, there is a "very large number" (a continuum whose measure is normalized to one) of identical individuals whose peculiarity is that they are sensible to their rank in the income hierarchy. Their utility grows with an increase in the share of participants whose incomes are lower than their own. In the first stage, referred to as the social competition game, individuals decide on their income, y. Their effort is an increasing function, $E(y)$, of their income. This effort function is assumed to be strictly convex such that $E(0) = 0$, and whose derivative $E'(y)$ satisfies $E'(0) = 0$ as well as $E'(\infty) = \infty$. The second stage is a signalling game in which the income distribution $F(y)$ is common knowledge. But, as incomes are private information, participants are led to dedicate part of their income to conspicuous consumption, c, in order to make their rank visible. The rest $(y - c)$ is dedicated to "ordinary consumption" which, contrary to conspicuous consumption, has an intrinsic value. Consider an individual whose rank is r in the interval $[0, 1]$. His/her utility, $u(y, r, c)$ is assumed to be

$$u(y, r, c) = -E(y) + (\alpha + \beta S(r))(y - c)$$

In this expression for utility, the "social multiplier", $S(.)$, is an increasing concave function of r $(S'(r) > 0, S"(r) \leq 0)$. In equilibrium, the rank of an individual with income y will coincide with the share, $F(y)$, of lower-income participants. The component $\alpha(y-c)$ $(\alpha > 0)$ reflects the fact that ordinary consumption also increases utility independently of the rank in the income scale. With no loss in generality, the top, $S(1)$, is set to one while the bottom, $S(0)$, is lower than one. Parameter β is a positive scalar. In the present section and in the next, the share (the measure of the set) of individuals with equal income is zero, whatever this income might be. Section 4 allows for mass points in the definition of $S(.)$.

The equilibrium of the signalling game is a relation, $c = c(y)$, between income and conspicuous consumption. From the second stage (signalling) equilibrium, one can deduce individuals' indirect (i.e. reduced) utility which only depends on their own income and the income distribution, $F(.)$. With these indirect utility functions, the first stage game becomes static.

Let us solve this signalling game. To that end, the analysis is restricted to the case in which the density, $F'(y)$, is positive on the range $[m, M]$. We show that any income distribution, $F(.)$, is associated with a single conspicuous consumption function, $c(.)$, which makes incomes visible. This means that an individual whose conspicuous consumption is c has the income $y = c^{-1}(c)$.

Let $G(c(z))$ denote the share of individuals whose positional consumption is lower than $c(z)$. Suppose that for each observed consumption, $c = c(y)$, society will "deduce" that the individual has an income of y. Assuming that $c(\cdot)$ is strictly increasing[2] the rank of an individual who decides on conspicuous consumption $c(z)$ coincides with the share of participants who have an income lower than z.[3] Since $F(.)$ is common knowledge, it follows that $G(c(z)) = F(z)$.

As incomes are given at this stage, the utility of an y-individual who chooses positional consumption $c(z)$ only depends on

$$V(z, y, c(.)) = [\alpha + \beta S(G(c(z)))] (y - c(z)) = [\alpha + \beta S(F(z))] (y - c(z)). \qquad (1)$$

Each unit of income spent on ordinary consumption, $(y - c)$, provides a utility of $\alpha + \beta S(F(z)$. The higher the consumption $c(z)$ of the positional good, for any given z, the less the individual will benefit from consuming the normal good. Thus the term $[\alpha + \beta S(F(z))] c(z)$ is a utility loss which represents the cost of mimicking the behavior of z-participants.

Consistency requires that $y-$individuals truthful, in the sense that their conspicuous consumption should be equal to $c(y)$. In other words, a (deviant) y-individual should not be prompted to mimic the behavior of some z-individual, whether z is

[2] *A priori*, to be a perfect signal, $c(.)$ only needs to be reversible. It could then be decreasing, but this case is easy to rule out.

[3] The hypothesis that $c'(\cdot) > 0$ implies that all z-individuals, with $z \neq y$, reveal their income correctly when deciding on their conspicuous consumption.

higher or lower than y. Therefore, the derivative of $V(.)$ with respect to z should be zero at $z = y$ for all y in the support of $F(\cdot)$:

$$\frac{\partial V(z, y, c(.))}{\partial z}|_{z=y} = \beta S'(G(c(y)))G'(c(y))c'(y)(y-c(y))-[\alpha+\beta S(G(c(y)))]c'(y) = 0 \tag{2}$$

Since $G(c(y))$ then coincides with $F(y)$ for all y in $[m, M]$, it follows that

$$\beta S'(F(y))F'(y)(y - c(y)) - [\alpha + \beta S(F(y))]c'(y) = 0 \tag{3}$$

The previous equation is referred to as the "marginal" truth-telling condition. Knowing that mimicking a higher ranked z-individual means consuming more of the positional good, the marginal utility loss, $[\alpha + \beta S(F(y))]c'(y)$, from increasing the positional consumption must equal the marginal benefit of y-individual from consuming the normal good, namely $\beta S'(F(y))F'(y)$, when $z = y$.
Using

$$\{[\alpha + \beta S(F(y))]\, c(y)\}' = \beta S'(F(y))F'(y)c(y) + [\alpha + \beta S(F(y))]c'(y) \tag{4}$$

the first-order condition at $z = y$ is rewritten as:

$$\{[\alpha + \beta S(F(y))]\, c(y)\}' = \beta S'(F(y))F'(y)y \tag{5}$$

From the first order condition of y-participants, we obtain that their utility loss, which reflects the cost of making their income visible, satisfies the condition:

$$[\alpha + \beta S(F(y))]c(y) = (\alpha + \beta S(0))c(m) + \beta \int_m^y S'(F(z))F'(z)zdz, \forall y \in (m, M]\,. \tag{6}$$

An interesting and intuitive point is that the cost of making income y observable depends on the incomes of lower-ranked agents. This is because the incentive of a low-ranked individual mimicking highly-ranked individuals is all the stronger the higher his/her income is. Consequently, his/her conspicuous consumption should grow with his/her income. But, the positional consumption of highly ranked individuals should be greater than that of low-ranked ones. This explains why the conspicuous consumption of an individual grows with the increase of income for lower-ranked participants.

Substituting (6) into the utility function, we can rewrite the latter as

$$V(z, y, F(.), c(m)) = [\alpha + \beta S(F(z))]\, y - \beta \int_m^z S'(F(t))F'(t)tdt - (\alpha + \beta S(0))c(m) \quad (7)$$

We see that

$$\frac{\partial V(z, y, .)}{\partial z} = \beta S'(F(z))F'(z)\,(y - z) \quad (8)$$

It emerges that this derivative is strictly positive for $z < y$ and strictly negative for $z > y$. This means that $z = y$ is a global maximum of the function for all y in $]m, M]$. In other words, telling the truth is optimal.

Further, writing $c(y)$ from (6) and differentiating, we obtain:

$$c'(y)\,[\alpha + \beta S(F(y))]^2 = \beta\,[\alpha + \beta S(F(y))]\, S'(F(y))F'(y)[y - c(y)]. \quad (9)$$

This proves that the consistency requirement, that $c'(y) > 0$, is fulfilled.

To conclude the derivation of the signalling equilibrium, we show that, in conformity with intuition, the positional consumption $c(m)$ of lower-income individuals ($y \leq m$) falls to zero. Suppose that the social belief is that this consumption is positive ($c(m) \geq 0$). From equation (8), we can deduce that individuals will not decide on higher conspicuous expenditure. As their rank (zero) cannot be lower, this implies that they will set this expenditure to zero. Consequently, to be consistent, the social belief should be that $c(m) = 0$. Conversely, individuals whose incomes are higher than M, who cannot improve their rank, decide on conspicuous consumption $c(M)$.[4]

To summarize, we can state the following Lemma.

Lemma 1. *Suppose that the social belief is that $y \leq m$ if $c = 0$, $y \geq M$ if $c = c(M)$ and that $y = z$* if *$c = c(z)$, where*

$$c(z) = \beta \frac{\int_m^z S'(F(t))F'(t)tdt}{\alpha + \beta S(F(z))}, \forall z \in (m, M]\,. \quad (10)$$

[4]In equilibrium, the (measure of the) set of individuals whose income is outside $[m, M]$ reduces to zero. Nonetheless, any income $y \geq 0$ should be associated with a reduced utility $U(y, .)$.

Then this consumption pattern forms a unique equilibrium for the signalling game.

3 Indirect utilities and equilibrium

3.1 KUJ as an outcome of GAS

Anticipating his/her second-stage rational decision (and so using (7) with $c(m) = 0$)), each individual derives an indirect (or reduced) utility function. Let us consider the benchmark case where $S(F(.)) = F(.)$. Under this assumption, agents' payoffs are given by

$$U(y, F(.)) = -E(y) + V(y, .) = -E(y) + \alpha y + \beta \int_m^y F'(z)(y - z)dz. \qquad (11)$$

It is worth comparing the previous expression for utility with the so-called cardinal version as proposed by Friedman and Ostrov (2008) (F0, henceforth). In this paper, the authors retain the following formula for social concerns

$$FO \equiv \gamma \int_0^x F'(z)(x - z)dz + (1 - \gamma) \int_x^\infty F'(z)(x - z)dz$$

with x being (ordinary) consumption and $F(.)$, the corresponding c.d.f. The positive parameter γ is lower or equal to one.

Suppose $\gamma = 1/2$ and let $\bar{x}$ denote the average consumption of the population. In this particular case, the FO criterion coincides with the usual KUJ version of relative concerns: $FO = KUJ = 1/2(x - \bar{x})$. In general, one can distinguish between two terms in the FO criterion. The first integral, which is positive, reflects a feeling of "pride", while the second integral translates a feeling of "envy". One interesting outcome is that, assuming that the social multiplier, $S(.)$,coincides with $F(.)$, the "pride" motivation ($\gamma = 1$)) (in terms of incomes) is a particular case of the first stage (indirect) utilities in our setting (see equation (11)). In other words, assuming that people are preoccupied with their rank in the income hierarchy, and that their conspicuous consumption make their rank visible, this gives rise to a static game in which individuals behave as if they were driven by the pure "pride" motivation in deciding on their income (i.e their effort). This outcome is close in spirit to Bilancini and Boncinelli (2014). In a marriage setup, these authors show that *allowing for asymmetric information leads to reduced preferences for social status that induce concern not only for one's own rank in the distribution of attributes, but also for how much higher or lower one's attribute is.*

3.2 Social competition equilibrium

We here solve the static reduced game without making use of the above specification of social concerns. In other words, the social multiplier $S(.)$ is a strictly increasing function, such that $0 < S(0) < 1$ and $S(1) = 1$. Using (7) with $c(m) = 0$, at the first stage of the social game, individuals' reduced utilities, $U(y,.) = -E(y) + V(y,.)$, can be written as

$$U(y, F(.)) = -E(y) + .\left[\alpha + \beta S(F(y))\right] y - \beta \int_m^y S'(F(z))F'(z)zdz \tag{12}$$

In the social competition stage participants decide on their income by maximizing $U(y,.)$ for a given distribution $F(.)$. A global equilibrium - i.e. an equilibrium of the static reduced game - can be defined as below.

Definition 1. *A global equilibrium is a distribution of incomes, $F^*(.)$, such that utility $U(y, F^*(.))$ is maximized with respect to y for all incomes in its support.*

One implication of this definition is that all individuals have the same utility in an equilibrium. This equilibrium would not otherwise be stable in the sense that some of the players would find it profitable to deviate. In other words, $U(y, F(.))$ should be a constant in the interval $[m, M]$ associated with $F(.)$, and lower than this constant outside this interval.

This equal-utility condition implies that the derivative of $U(y, F(.)$ with respect to y is zero in the open interval $]m, M[$, that is

$$-E'(y) + \alpha + \beta S(F(y)) = 0$$

We obtain that in the range $]m, M[$, the equilibrium distribution is deduced from

$$S(F(y)) = \frac{E'(y) - \alpha}{\beta} \tag{13}$$

Let us now study the way in which the bounds m and M are set. Consider first the lower bound, m. For $y < m$, the derivative of $U(.)$ with respect to y, $-E'(y) + \alpha + \beta S(0)$, should be positive or zero. Otherwise, m would not be the lowest income. Symmetrically, for $y > m$, this derivative should be negative (since $S(F(y)) > 0$). Consequently, we should have $E'(m) = \alpha +, \beta S(0)$. Since $E'(0) = 0$ and $E'(\infty) = \infty$, this equation has a single solution, m^*.

Let us turn to the higher bound M. Above M, the derivative, $-E'(y)+\alpha+\beta$, should be negative (as $S(F(y)) = 1$ for $y > M$). Below M, this derivative should be positive (since $S(F(y)) < 1$). Consequently, the higher bound is the solution to $E'(y) = \alpha+\beta$. Knowing the assumptions we made concerning the derivative of $E(y)$, the previous equation possesses one single solution, M^*.

It follows that

Proposition 1. *The first stage game has a single equilibrium $F^*(.)$ such that $F^*(y) = S^{-1}((\frac{E'(y)-\alpha}{\beta}))$ in the support $[m^*, M^*]$ with m^* being the solution to $E'(y) = \alpha+\beta S(0)$ and M^* the solution to $E'(y) = \alpha + \beta$.*

Note that this equilibrium is easy to derive. The bounds m and M are determined by using the fact that $F(m) = 0$ and $F(M) = 1$ respectively. Between the two extremes $F^*(y)$ is computed so as to make utility constant. It is worth mentioning that, regarding the existence and uniqueness of market equilibrium, the concavity of GAS multiplier $S(r)$ is not required.

4 Welfare

Hopkins and Korienko (2004) find that for any income distribution, the signalling equilibrium is inefficient. The reason for this is that in their setting, conspicuous consumption is not a purely positional good. Like non-positional consumption, conspicuous consumption also has an intrinsic value. In this case, laissez faire is dominated by the conspicuous consumption that people would choose in the absence of social concerns, which the authors refer to as the cooperative situation. As it is an increasing function of income, cooperative conspicuous consumption reveal incomes. In our setting where conspicuous consumption is a pure signal, their "cooperative" level is zero for all incomes. Consequently the inefficiency that Hopkins and Korienko emphasize does not make sense in our model. In our set up, we are led to treat the signalling equilibrium as a constraint. In other words, the cost of making incomes visible, associated with the second stage equilibrium, are seen as unavoidable.

4.1 Utilitarian (in)efficiency

Following the utilitarian principle, our criterion for social efficiency is the sum of individuals utilities, denoted by Σ.

$$\Sigma(F(.)) = \int_m^M U(y, F(.))F'(y)dy \tag{14}$$

A simple way of comparing the laissez-faire equilibrium, $F^*(.)$, with a social optimum is by distributing the agents uniformly and continuously on the scale $[0,1]$. Since the distribution is uniform, the position, x, of an individual i in this interval, is also the share of players whose positions are lower than x in this segment. Let $y(x)$ be the income of x-individuals (individuals of position x). One can see that any (strictly) increasing and continuous income function, $y(x)$, can be mapped on to an increasing and continuous distribution function $F(.) = y^{-1}(.)$, and reciprocally.[5] In this case, the position x of an individual coincides with his/her rank $r = F(y(x)) = x$. Next, suppose that $y(x)$ is zero on the range $[0, x_1]$ and strictly increasing above x_1. One can see that the corresponding distribution function has a mass point in zero of frequency x_1. On the other hand, suppose that $]A, B[$ is a hole of $F(.)$. Since all incomes in this interval have the same rank $F(B) = F(A) = r$, the income function is discontinuous at $x = r$ with $y(r^+) = B > y(r) = A$. More generally, any non-decreasing income function, $y(x)$, defined on $[0,1]$, can be mapped into a distribution function, $F(y)$.

It results that the social utility, Σ, can be rewritten as a function of $[y(.)]_0^1$.

$$\Sigma(y(.)) = \int_0^1 U(x, y(x), y(.))dx$$

or

[5] This results from the fact that for any random variable, X, of c.d.f. $F(.)$, the random variable $Y = F(X)$ is uniformly distributed over the segment $(0, 1)$.

$$\Sigma(y(.)) = \int_0^1 [-E(y(x)) + \alpha y(x) + \beta S(x)y(x) - \beta \int_0^x S'(t)y(t)dt]dx \tag{15}$$

From this, it can be deduced that when deciding on the income $y(x)$ of x-individuals, the planner maximizes

$$\mu(x, y(x), [y(.)]_x^1) = -E(y(x)) + \alpha y(x) + \beta S(x)y(x) - \beta \int_x^1 S'(x)y(x)dt \tag{16}$$

with respect to $y(x)$, where x is given.

It is worth noticing that the social planner takes into account the fact that an increase in $y(x)$ lowers the utility of individuals whose ranks are higher $(1 > t > x)$ by raising their signalling costs.

As a consequence, the derivative of $\mu(x, y(x), [y(.)]_x^1)$ with respect to $y(x)$ is

$$\frac{\partial \mu(x, y(x), [y(.)]_x^1)}{\partial y(x)} = -E'(y(x)) + \alpha + \beta S(x) - \beta(1-x)S'(x) \tag{17}$$

In this derivative, the term $(1-x)S'(x)$ measures the positive impact of an increase of $y(x)$ on the signalling costs of higher ranked individuals, while $\alpha + \beta S(x)$ is the positive impact on individuals whose position is x. From the concavity of $S(.)$ $(S"(.) \leq 0)$, it follows that the impact on gross welfare $\Upsilon(x) = \alpha + \beta S(x) - \beta(1-x)S'(x)$ is strictly increasing in x.

For $x = 0$, the derivative, $\partial \mu(x, y(x), .)/\partial y(x)$, will be negative if $\alpha + \beta S(0) < \beta S'(0)$ and, positive if $\alpha + \beta S(0) \geq \beta S'(0)$. Remember that $E(0) = E'(0) = 0$. Let us first treat the case where $\alpha + \beta S(0) \geq S'(0)$. As $\Upsilon(x)$ is strictly increasing, we deduce that the derivative of $\mu(x, y(x), .)$ with respect to $y(x)$ for $y(x) = 0$ is strictly positive for all x in $]0, 1]$. In this situation a social optimum, denoted by $y^s(x)$, should satisfy the following necessary and sufficient condition for all x in the interval $[0, 1]$:

$$\frac{\partial \mu(x, y(x), [y(.)]_x^1)}{\partial y(x)} = 0 = -E'(y(x)) + \alpha + \beta S(x) - \beta(1-x)S'(x)$$

It can be verified that the optimal income function $y^s(x)$ is continuous and strictly increasing on $[0, 1]$. This means that the rank of an $y^s(x)$-individual coincides with his/her position, x, on the unit length segment. Consequently the previous (social) optimality condition can be rewritten as

$$E'(y(r)) = \alpha + \beta S(r) - \beta(1-r)S'(r) \tag{18}$$

and the optimal distribution, $F^s(y)$, is the reciprocal of $y^s(r)$.

The social optimum $F^s(.)$ is strictly increasing. Its lower bound, m^s, is the solution to $E'(m) = \alpha + \beta S(0) - \beta S'(0)$, while its higher bound is the solution to $E'(M) = \alpha + \beta$. Notice that the social optimum has the same higher bound as a laissez-faire equilibrium. The intuition behind this is that an increase in $y(1)$ does not have an impact on the utility of lower ranked individuals (by increasing signalling costs). On the contrary, the minimum m^s is lower than m^* because a social planner knows that an increase in $y(0)$ has a negative impact upon the utility of all individuals.

Let $y^*(.)$ denote the reciprocal of market equilibrium $F^*(.)$. This income function satisfies

$$E'(y^*(r)) = \alpha + \beta S(r) \tag{19}$$

Since $E(y)$ is strictly convex, comparing equations (18) and (19) shows that the laissez-faire income function, $y^*(r)$, is above the socially optimal income function, $y^s(r)$. Written in words, this means that laissez-faire generates incomes that are too high. Once again, this is not surprising, since when deciding on his/her income, an individual does not take into account that an increase in his/her income reduces the utility of all higher-ranked participants by increasing their signalling costs.

Let us now show that this inefficiency also holds when $\alpha + \beta S(0) < \beta S'(0)$; hence where an increase of lower incomes strongly affects the signalling costs of other participants.

The above analysis suggests that in this regime the (socially optimal) distribution function will have a mass point in zero. Indeed, such a mass point is the means of increasing the rank of strictly positive incomes in the hierarchy. To check for this intuition we need to determine the equilibrium of the signalling game when the distribution $F(.)$ has a mass point in zero, meaning that a positive share of the population, $\phi(> 0)$ has the same income, 0. For simplicity, it is assumed that individuals are only concerned about strictly lower incomes. In other words, having equals adds nothing to their social position. Consequently, all 0-income participants have the same social rank $r = 0 = F(0) - \phi$.[6]

For obvious reasons, 0-individuals all have the same conspicuous consumption $c(0) = 0$. As their income is zero, they cannot announce a higher social rank than zero.[7] Regarding higher-income individuals, it is easy to see that their conspicuous consumptions are still derived from the truth-telling condition (equation (3)).[8]

The existence (and uniqueness) of the signalling equilibrium clearly holds true in the presence of a mass point in zero. Consequently, we can consider the case in which the social planner decides on a mass point in zero. In this circumstance, individuals with a (strictly) positive income (in proportion $(1-\phi)$) are distributed uniformly over the segment $[\phi, 1]$. Formally, we have $y(x) = 0$ for all x in the interval $[0, \phi]$. The share ϕ is set in such a way that (strictly) positive incomes increase social welfare. This requires that the derivative

$$\frac{\partial \mu(r, y(r), [y(.)]_{\phi}^{1})}{\partial y(r)} = -E'(y(r)) + \alpha + \beta S(r) - \beta(1-r)S'(r)$$

be strictly positive for $r > \phi$ and $y(r) = 0$. A necessary and sufficient condition is that $-E'(0) + \alpha + \beta S(0) - \beta(1-\phi)S'(0) = 0$. Consequently, the utilitarian planner decides on $\phi^s = 1 - \frac{\alpha + \beta S(0)}{\beta S'(0)}$. Below ϕ^s, (socially) optimal incomes, $y(x)$, are zero, hence lower than equilibrium incomes. Above $r = \phi^s$, incomes $y^s(r)$ are still derived from

$$E'(y(r)) = \alpha + \beta S(r) - \beta(1-r)S'(r)$$

which implies that they also are lower than equilibrium incomes.

To summarize, we have proved the following proposition:

Proposition 2. *According to a utilitarian planner, the laissez-faire income function, $y^*(.)$, is above the socially optimal income function, $y^s(.)$.*

[6]Assuming that $r = F(0) - \rho\phi$ with $0 < \rho < 1$ does not affect the analysis.

[7]Notice that, with a mass point $Y > 0$, the truth-telling condition would generate a discontinuity of $c(y)$ in Y. This discontinuity ($c(Y^+) > c(Y)$) would compensate for the discrete increase of the rank $(F(Y) - F(Y^-))$. See the appendix.

[8]Their utility is strictly higher than zero, hence strictly higher than 0-individuals' utility.

We already know the origin of this inefficiency. Individuals do not take into account the effect of their decision on the signalling costs of higher-ranked participants.

Laissez-faire equilibrium is not a utilitarian social optimum. On the other hand one can see that, although agents are homogenous, this utilitarian optimum is not egalitarian. In this utilitarian situation high-ranked individuals enjoy higher utilities than low-ranked ones. As already noted, the intuition is that an increase in the income of an individual (i) (negatively) affects high-ranked participants the more, the lower the the rank of this individual (i). An expressive consequence is that a utilitarian planner should assign a zero income to part of the population in order that individuals with positive incomes reach a high social rank, thus increasing the (social) marginal return to their incomes. This raises the following issue: how should a social planner behave if he is not only utilitarian, but also egalitarian?

4.2 Egalitarian and Pareto efficiency

If egalitarian, the planner maximizes $\Sigma(y(.))$ subject to the constraint that all utilities $U(x, y(x), [y(.)]_0^1)$ have some constant (endogenous) level, U^s. To solve the problem of this egalitarian planner, we first observe that the solution, $y^*(0)$, to $-E'(y) + \alpha + \beta S(0) = 0$ generates a utility level, $U^*(0)$ for 0-individuals which cannot be increased. So the question is: Can higher-ranked participants reach this utility level? We already know the answer to this question. Laissez-faire equilibrium is so constructed that all individuals enjoy the same utility level $U^* = U^*(0)$. Consequently, $U^s = U^*$ and we have proved

Proposition 3. *According to an egalitarian planner, laissez-faire equilibrium is socially optimal.*

It can be noted that this efficiency result is particularly relevant with identical agents. It also shows how perceived inequality (income inequality) can coincide with true equality (utility equality).

The fact that laissez-faire equilibrium is not a social optimum (according to the utilitarian criterion) suggests that it is not Pareto efficient. The formal proof consists of studying the effect on utilities of a small variation Δ in all incomes in the neighborhood of equilibrium $F^*(.)$. Since the income level associated with rank r becomes

$z(r) \equiv y^*(r) + \Delta$, with $y^*(r)$ being the reciprocal of $F^*(.)$, utilities $U(r, z(r), z(.))$ can be written as

$$U(r, z(r), z(.)) = -E(z(r)) + (\alpha + \beta S(r))z(r) - \beta \int_0^r S(t)z'(t)dt$$

Because the derivative $dz(r)/dr$ coincides with $dy^*(r)/dr$, the variation in $U(r, .)$, for all r in $[0, 1]$, is close to

$$[-E'(y(r)) + \alpha + \beta S(r) - \beta S(r)]\Delta$$

Now, from the equilibrium equation (Proposition 1), we deduce that, in the neighborhood of laissez-faire, the previous expression reduces to: $-\beta S(r)\Delta$. This shows that a small decrease in all incomes ($\Delta < 0$) raises all utilities $U(r, .)$. Considering the case where all individuals (i) keep the same rank ($r(i)$) in $[0, 1]$, this proves that equilibrium, $F^*(.)$, is not a Pareto optimum. Lowering all incomes raises all utilities $U(r(i), .)$. The interpretation of this result follows the same line as our comment on Proposition 2.

To summarize

Proposition 4. *According to a Paretian planner, laissez-faire equilibrium is inefficient.*

This result emphasizes the price that is paid for equality in an environment where individuals are homogenous but are preoccupied with their social status.

4.3 Information and welfare

As already mentioned, earlier work usually focus on the determination and on the efficiency of the signalling game. In particular, Hopkins and Horienko (2004) show that an appropriate tax on positional consumption is capable of making laissez-faire coincide with the cooperative case which corresponds to perfect information. When incomes are exogenous, perfect information obviously improves welfare. Does this hold true when incomes are endogenous? Let us compare the imperfect information equilibrium (IIE) with the perfect information equilibrium (PIE). When all incomes are observable, the status game has a single stage in which players maximize their utility, $\hat{U}(y, F(.)) = -E(y) + (\alpha + \beta S(F(y)))y$, for a given distribution $F(.)$. An equilibrium is a distribution $\hat{F}(.)$ such that $\hat{U}(y, \hat{F}(y))$ is maximized for any income y in its support. Following the same line of reasoning as in the proof of Proposition 1, the condition for equal utilities implies that, for all r in $]0,1[$, the associated (PIE) income function, denoted by $\hat{y}(r)$, satisfies[9]

$$-E'(y(r))y'(r) + (\alpha + \beta S(r))y'(r) + \beta S'(r)y(r) = 0 \tag{20}$$

or, as $1/y'(r) = F'(y(r))$,

$$E'(y(r)) = \alpha + S(r) + S'(r)F'(y(r))y(r) \tag{21}$$

On the other hand, we know that, for all r in $]0,1[$, the (IIE) income function, $y^*(r)$, satisfies $E'(y(r)) = \alpha + \beta S(r)$

As the effort function is strictly convex, we obtain the result that perfect information increases the income $y(r)$ for almost all r. The reason for this is that, in the absence of signalling costs, the price paid for an income increase amounts to an increase in effort. Consequently, perfect information tends to degrade (utilitarian) welfare by strengthening the rat-race effect in the social competition game. On the other hand, imperfect information creates signalling costs which are dead weight losses. We can conclude that perfect information is not necessarily better for welfare.

5 A more general setting

So far we have made use of restrictive assumptions which rendered the analysis quite simple. We now relax these assumptions and show that results are left unchanged. Which means that the reader can focus on earlier developments and ignore any further implications.

The extension that we develop here bears on the assumed properties of the income distribution, $F(.)$, where we allow for holes and mass points. There are different reasons for this extension. First, what we actually showed above is that, assuming that $F(.)$ is continuous with a connected support, then the equilibrium distribution, F^*, does possess such properties. This is not sufficient. There may be an equilibrium with holes and/or mass points. Next, to be perfect in Selten's sense the equilibrium

[9]See Gavrel and Rebiere (2015).

of the signalling sub-game should be defined for any income distribution. Finally, how is the utilitarian optimum affected?

To address these issues we study the formation of a signalling equilibrium in the presence of holes and discontinuities.

5.1 Signalling equilibrium

The appendix of the text proves the following Lemma.

Lemma 2. *Any distribution, $F(.)$, is associated with a single signalling equilibrium, $c(., F(.))$. Any positive mass point, $A > 0$, is mapped on to a discontinuity such that $c(A^+, F(.)) > c(A, F(.))$. Any hole, $]B, C[$, translates into the constancy of conspicuous consumption $c(.)$ in this hole.*

What makes these statements intuitive is that conspicuous consumption, $c(y, F(.))$, actually reflects the rank in the income scale of an individual who earns income y. Knowing that individuals whose income lies in a hole $]A, B[$ have the same rank $R = F(B) = F(A)$, the truth-telling condition implies that all these individuals have the same conspicuous consumption $c(B) = c(A)$. It is worth noting that, if expressed as a function of the rank, the derivative of positional consumption, $c'(r)$, jumps upwards on the right hand of R ($c'(R^+) > c'(R)$). The reason is that the impact on utility of an increase in the rank is the stronger, higher the income level. As a consequence, in signalling equilibrium the marginal cost of such an increase should increase with the income level.

The reason why a positive mass point A of frequency ϕ generates a discontinuity in conspicuous consumption is easy to understand. Assuming that "having equals does not add anything to individuals", the social multiplier, $S(.)$, jumps upwards in A by the amount, $S(F(A)) - S((F(A) - \phi))$. The truth-telling condition then imposes that conspicuous consumption also jumps upwards in A to discourage A-individuals from imitating higher-income participants.

Once the existence and uniqueness of the signalling equilibrium has been set, we can study the determination of a global equilibrium (reduced game of stage 1) as well as the determination of a utilitarian optimum.

5.2 Global equilibrium and utilitarian optimum

We first show that the condition for global equilibrium excludes holes as well as mass points in (the support of) income distribution. To that end, we make use of the following lemma.

Lemma 3. *In any interval I where $F(.)$ is either strictly increasing or constant everywhere, the derivative of stage-1 utility, $U(y,.)$, with respect to y reduces to*

$$-E'(y) + \alpha + \beta S(F(y))$$

The formal proof is provided in Appendix A. It is straightforward if I is a hole, since $c(.)$ is constant in this case. If I belongs to the support of $F(.)$, the lemma is derived from the "marginal" truth-telling condition by applying the *enveloppe* theorem.

Lemma 3 plays a key role in ruling out mass points and holes. Let us here illustrate this role by considering a mass point $A > 0$ of frequency $\phi > 0$. We know from the analysis above that in such a case, positional consumption is discontinuous in A with $c(A^+) > c(A)$. From Lemma 3, we deduce that the two inequalities below should be jointly satisfied.

$$-E'(A) + \alpha + \beta S(F(A)) \leq 0$$

and

$$-E'(A) + \alpha + \beta S(F(A) - \phi) \geq 0$$

The first (second) inequality comes the fact that stage 1 utility should not increase on the right (left) hand of mass point A. As the social rank jumps upwards in A ($r(A^+) - r(A) = \phi$), these inequalities are a contradiction.

Appendix A also shows how holes in the income distribution are excluded. As a consequence, in global equilibrium, income distribution is strictly increasing in an interval $[m, M]$ with $F(y) = 0$ for $y \leq m$ and $F(y) = 1$, for $y \geq M$. In other words, $F(.)$ necessarily possesses the assumed properties in global equilibrium. To summarize, we can state the following

Proposition 5. Proposition 1 *includes the case where the support of the income distribution is not necessarily connected and where it possibly contains mass points.*

As for the utilitarian optimum, this extension is straightforward. This is because, as explained in the determination of the utilitarian optimum, any mass point is mapped on to the constancy of the income function $y(.)$ in some sub-interval of unit length segment. It is is easy to show that a hole of $F(.)$ translates into a discontinuity of the income function. Indeed, suppose $]A, B[$ is a hole with $F(A) = F(B) = R$. On the right hand side of R, the income function $y(r)$ should be strictly higher than income B whereas, on the left hand side, it should strictly lower than income A. These two properties create constraints on the planner's problem when this problem is formulated as the selection of an optimal income function. We then obtain that Proposition 2 also extends to the case where the support of income distribution is not necessarily connected and possibly contains mass points different from zero. The brief proof we provide for this extension is very laconic. To make it more intuitive, Appendix C shows why positive mass points necessarily degrade welfare.

6 Conclusion

This paper accounted for two behavioral hypothesis by building a two-stage status game. The main insights are as follows. GAS behavior, the desire for advantageous inequality, gives rise to income inequalities whether incomes are common knowledge, as in Gavrel and Rebiere (2015), or private information, as in the present paper. The GAS attitude generates equilibrium income dispersion in the same way as non-sequential search induces equilibrium price dispersion in Burdett and Judd (1983). Next, the "pure pride" version of social concern (Friedman and Ostroy, 2008) can be seen as an outcome of the GAS attitude in the signalling sub-game. This provides an interpretation for the assumption of cardinal social preferences. The average income in the population of lower-ranked individuals coincides with the cost of making incomes visible. Moreover, regarding welfare, market equilibrium is not a utilitarian optimum because individuals do not internalize the impact of their income upon the signalling costs of higher-ranked participants. But although it is not a Pareto optimum, laissez-faire equilibrium is an egalitarian social optimum.

An interesting issue is the extent to which common knowledge of individuals' incomes is socially desirable. On the one hand, public information regarding everybody's income reduces signalling costs (i.e. conspicuous consumptions) to zero. As these costs are a dead-weight loss, this tends to improve welfare. On the other hand, these costs tend to lower incomes which are too high relative to a utilitarian social optimum. Taking a political economy perspective, this trade-off might help explain why information on incomes is public in some countries, like Norway, but confidential in others, like France.

Gavrel and Rebiere (2015) shows that, assuming that anyone's rank in the income scale is known of anybody, (pure) GAS equilibrium fairly well accounts for the distribution of observed wages (in France). An interesting line for empirical investigations would be to compare the empirical potential of the GAS hypothesis: whether information is perfect or imperfect. Knowing that with imperfect information GAS generates Pride and that the Pride equilibrium is much simpler than the pure GAS equilibrium, we can surmise that pure GAS is likely to be a better explanation of the empirical intermediate wage distribution (for France).

To conclude, we would like to note that our analysis could be translated to other fields, like industrial organization. For instance, consider firms who first decide on the quality (the rank) of the goods they supply. Suppose then that, in a second stage, they must let customers know this quality. A natural and practically relevant assumption would be that these firms spend (conspicuous) amounts on any item, provided that this expenditure is known to customers. This example shows that ordinal concerns are not incompatible with pure rationality. Ordinal concerns can be instrumental. Notice that, similar to Bilancini and Boncinelli (2014), in the presence of these advertising (signalling) costs, firm's reduced profits (deduced from the signalling sub-game) would have a cardinal form. Profits would decrease with an increase in the quality of lower-ranked goods which raises signalling costs. To some extent, researchers face the same type of situation: producing papers whose value (rank) depends on their effort (among other skills), and then making this value visible. Indeed, if the scholar is not sure that his/her paper is good, he/she will not spend as much money in traveling costs and other apparently futile expenses.

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Appendix A. A (more) general model: Game equilibrium. Proof of Lemma 2, Lemma 3 and Proposition 5

We here deal with the (more) general case where the support of $F(.)$ possibly can contain mass points and holes. In any interval of its support, $F(.)$ is assumed to have a strictly positive derivative. As a first step, we show that the existence and uniqueness of the signalling equilibrium holds true for any distribution $F(.)$ of this general class.

We also prove that holes and mass points are excluded in game equilibrium. This means that the assumptions made above can be relaxed without affecting the results established in the text. In addition, the extension to holes and mass points means that the two-stage game is well defined, in the sense that any strategy y_i of an individual i can be associated with a utility $U(y_i, .)$ for any strategies no the part of other individuals (i.e. for any function $F(.)$, as required in a perfect equilibrium.

Different cases must be distinguished. In each case, we show how the the truth-telling condition determines conspicuous consumptions, point "a". In other words, we provide the proof of Lemma 2. Then, in each case, we ask the following question. Is this case compatible with equilibrium in stage 1? The answer, which is always negative, is given in point "b". This proves Proposition 5.[10] In Appendix B, we ask the same question concerning a utilitarian optimum.

To that end, we begin with proving Lemma 3, which is used in point b.

Lemma 3. *In any interval I in which $F(.)$ is either strictly increasing or constant, the derivative of stage-1 utility, $U(y, .)$, with respect to y reduces to*

$$-E'(y) + \alpha + \beta S(F(y))$$

Indeed, suppose that $F(.)$ is constant in an interval I. Then, all individuals in this interval have the same rank, R, hence the same conspicuous consumption, $c(R)$. Consequently, their utility $U(y, .)$ is

$$U(y, .) = -E(y) + (\alpha + \beta S(R))(y - c(R))$$

This proves Lemma 3 for this first case. Suppose now that $F(.)$ is strictly increasing in I. The marginal truth-telling condition then requires that the derivative of $V(z, y, c(.), F(.))$ with respect to z is zero in $z = y$. Consequently

$$\beta S'(F(y)F'(y))(y - c(y)) - (\alpha + \beta S(F(y)))c'(y) = 0$$

This proves Lemma 3.

Case 1. Suppose $F(.)$ is strictly increasing in a left hand neighborhood of A, constant in the interval $I =]A, B[$ with $B > A > 0$, and strictly increasing in a right hand neighborhood of B.

[10] We acknowledge that this exposition of proofs is not usual.

-a) We first observe that what individuals make visible is not their income but their rank. All incomes in I have the same rank $R = F(A) = F(B)$. This imposes that all individuals in this interval have the same conspicuous consumption.
Below A and above B, that is below R and above R, the derivative of $V(z, y)$ with respect to z should be equal to zero for $z = y$. This implies that

$$\beta S'(R)(B - c(R)) - (\alpha + \beta S(R))c'(B^+) = 0$$

and

$$\beta S'(R)(A - c(R)) - (\alpha + \beta S(R))c'(A^-) = 0$$

Notice that this means that $c'(R^+)$ is strictly greater than $c'(R)$. The reason is that the incentive to cheat is stronger with a higher income. In addition, these truth-telling conditions ensure that an individual with an income y in I, whose rank is R, will not be prompted to cheat. From this, it follows that we can define a signalling equilibrium in the presence of the hole I, which also means that a reduced utility $U(.)$ can be associated with any y in I.
-b) From the previous equations, knowing that $E(.)$ is strictly convex, we can deduce that

$$-E'(A) + \alpha + \beta S(R) > -E'(B) + \alpha + \beta S(R)$$

This proves that the hole, I, is not not compatible with the definition of an equilibrium in stage 1.
Case 2. We now turn to mass points. Suppose $F(.)$ has a mass point $A > 0$ of frequency $\phi > 0$.
-a) All individuals with income A have the same rank, $F(A) - \phi$ whereas individuals with incomes $y > A$ have the rank $F(y) \geq F(A)$. The truth-telling condition requires that A-individuals do not mimic individuals with higher incomes and vice-versa. This implies that, for all $y > A$,

$$V(A, A, .) \geq V(y, A, .)$$

and, reciprocally

$$V(y, y, .) \geq V(A, y, .)$$

$S(.)$ is continuous. And the same holds for $c(.)$ on both sides of A.[11] Thus, as y goes to A^+, utility $V(y, y, .)$ goes to utility $(\alpha + \beta S(F(A)))(A - c(A^+))$, while utility $V(A, y, .)$ goes to $(\alpha + \beta S(F(A) - \phi))(A - c(A)) = V(A, A, .)$ and utility $V(y, A, .)$ goes to $(\alpha + \beta S(F(A)))(A - c(A^+))$. From this, we deduce

$$V(A, A, .) = (\alpha + \beta S(F(A) - \phi))(A - c(A)) \geq (\alpha + \beta S(F(A)))(A - c(A^+))$$

and,

$$(\alpha + \beta S(F(A)))(A - c(A^+)) \geq (\alpha + \beta S((F(A) - \phi))(A - c(A)) = V(A, A, .)$$

It follows that

$$(\alpha + \beta S(F(A) - \phi))(A - c(A)) = (\alpha + \beta S(F(A)))(A - c(A^+))$$

In words the discontinuity of $F(.)$ creates a discontinuity of $c(.)$. The reason is that the discrete increase in conspicuous consumption ($c(A^+) - c(A) > 0$) compensates for the upwards jump of the rank ($F(A) - (F(A) - \phi) = \phi$). This shows how (strictly) positive mass points affect signalling equilibrium.

-b) Let us now turn to stage 1. $E'(.)$ and $S(.)$ are continuous. From Lemma 2, we deduce that, as y goes to A^+, the derivative of stage-1 utility, $U(y, .)$, with respect to y, goes to

$$-E'(A) + \alpha + \beta S(F(A))$$

As a consequence of the definition of an equilibrium, $F^*(.)$, this limit should satisfy

$$-E'(A) + \alpha + \beta S(F(A)) \leq 0$$

Similarly, since $A > 0$, we deduce

$$-E'(A) + \alpha + \beta S((F(A) - \phi)) \geq 0$$

[11] What follows also proves this statement.

As $\phi > 0$, the two previous inequalities are contradictory. This proves that $F^*(.)$ cannot have strictly positive mass points.

Case 3. Suppose that 0 is a mass point of frequency $\phi > 0$.

-a) All individuals with income 0 have the same rank, 0, whereas individuals with incomes $y > 0$ have the rank $F(y) \geq \phi$. Since their income is zero, 0-individuals cannot mimic individuals with higher incomes. Consequently, $c(y)$ is continuous in 0. As y goes to zero, $c(y)$ goes to $c(0) = 0$.

-b) In the first stage, income 0 should be a maximum of $U(y, F(.))$. Therefore, the derivative of $U(y, .)$ with respect to y should be negative (or zero) in a right hand neighborhood of 0, denoted by I. Using Lemma 2, we deduce that, for all y in I,

$$-E'(y) + \alpha + \beta S(F(y)) \leq 0$$

As y goes to 0^+, the previous derivative goes to

$$-E'(0) + \alpha + \beta S(\phi) = \alpha + \beta S(\phi) > 0$$

This contradiction proves that an equilibrium at stage 1 cannot have a mass point in zero.

From the analysis above, it emerges that, in equilibrium, the distribution of incomes, $F(.)$, has neither holes nor mass points. Hence $F^*(.)$ is strictly increasing in an interval $[m, M]$[12]. What was an assumption in the text turns out to be the main result of Appendix A. This proves Lemmz 2 and Proposition 5.

Appendix B. A (more) general model: utilitarian optimum

The purpose of Appendix B is to extend the determination of a utilitarian optimum to the presence of holes and mass points. A simple but somewhat laconic proof amounts to observing that mass points (different from zero) and holes create constraints on the planner's problem when this problem is formulated as selecting an optimal income function $y(x)$ on the segment $[0, 1]$. We believe that a more expressive proof would

[12]Since $E'(\infty) = \infty$, one can easily see that the support has an upward bound.

be welcome. For the sake of brevity, we restrict ourselves to mass points. We show that strictly positive mass points are not compatible with social optimality.

So, let us suppose that $F(.)$ has a mass point $A > 0$ of frequency $\phi > 0$. A first subcase is $F(A) = 1$. Since $A > 0$, we can deduce that the utility of individuals whose income is A, $U(A) = -E(A) + [\alpha + \beta S((1-\phi))][A - c(A)]$, is strictly positive. This implies that $-E'(A) + [\alpha + \beta S((1-\phi))][1 - c'(A)] = 0$. Notice that, from Appendix A, we know that a mass point A generates a discontinuity in $c(.)$. This does not mean however that $c(A)$ is not differentiable with respect to A as a change in A affects $c(A)$ as well as $c(A^+)$. Let B denote an income level such that $B > A$. We claim that redistributing individuals between incomes A and B improves welfare for B close to A. Let $b\phi$ $(1 \geq b \geq 0)$ be the share of B-individuals in total population (normalized to one) - that is, the frequency of mass point B - and, $(1-b)\phi$, be the share of A-individuals. The contribution of mass points A and B to social welfare, $W(b, B) \equiv (1-b)\phi U(A) + b\phi U(B)$, is

$$W(b, B) = (1-b)\phi[-E(A)+(\alpha+\beta S((1-\phi)))(A-c(A))]+b\phi[-E(B)+(\alpha+\beta S((1-b\phi)))(B-c(B))]$$

A important to point to notice here is that $c(A)$ does not depend on B. Indeed, according to the truth-telling condition, the determination of $c(y)$ is backwards.[13] In contrast, $c(B)$ actually depends on B (and b). The reason is that the truth-telling condition[14] implies that

$$V(B, A, .) = [\alpha + \beta S((1-b\phi))](A - c(B)) = [\alpha + \beta S((1-\phi))](A - c(A)) = V(A, A, .)$$

The previous relation determines $B - c(B)$ which depends on B and b:

$$[\alpha+\beta S((1-b\phi))](A-c(B)) = [\alpha+\beta S((1-b\phi))](B-c(B))-[\alpha+\beta S((1-b\phi))](B-A)$$

$$= [\alpha + \beta S((1-\phi))](A - c(A))$$

[13]More precisely, this means that $c(y)$ only depends on $[F(z)]_0^y$. The conspicuous consumption associated with income y is not affected by higher incomes.

[14]See Appendix A.

We deduce

$$[\alpha+\beta S((1-b\phi))](B-c(B)) = [\alpha+\beta S((1-\phi))](A-c(A))+[\alpha+\beta S((1-b\phi))](B-A)$$

Substitution into $W(.)$ yields

$$W(b,B) = \phi[-E(A)+(\alpha+\beta S((1-\phi)))(A-c(A))]-b\phi[E(B)-E(A)]+b\phi[\alpha+\beta S((1-b\phi))](B-A)$$

Consider the derivative of $W(b,B)$ with respect to B, for $B = A$:

$$\frac{\partial W(b,B)}{\partial B} = b\phi[-E'(A) + \alpha + \beta S((1-b\phi))]$$

Since $S((1-b\phi)) > S((1-\phi))$ for $0 < b < 1$, it follows that, for $0 < b < 1$,

$$b\phi[-E'(A) + \alpha + \beta S((1-b\phi))] > b\phi[-E'(A) + \alpha + \beta S((1-\phi))]$$

From the optimality of A, we derive that, for all b in $[0,1]$,

$$-E'(A) + \alpha + \beta S((1-\phi)) \geq -E'(A) + [\alpha + \beta S((1-\phi))](1-c'(A)) = 0$$

This implies that, for $0 < b < 1$,

$$\frac{\partial W(b,B)}{\partial B} > 0$$

This means that a mass point $A > 0$ such that $F(A) = 1$ cannot be a social optimum. "Smoothing" the support of $F(.)$ improves social welfare. Following the same line of reasoning, it can be proved that other mass points $A > 0$ are not compatible with a social optimum. The same holds for holes in the support of $F(.)$ which are mapped on to discontinuities of the income function, $y(x)$.